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HYBRIDISATION OF DISCONTINUOUS GALERKIN METHODS FOR SHOCK CAPTURING IN SCALE RESOLVING SIMULATIONS

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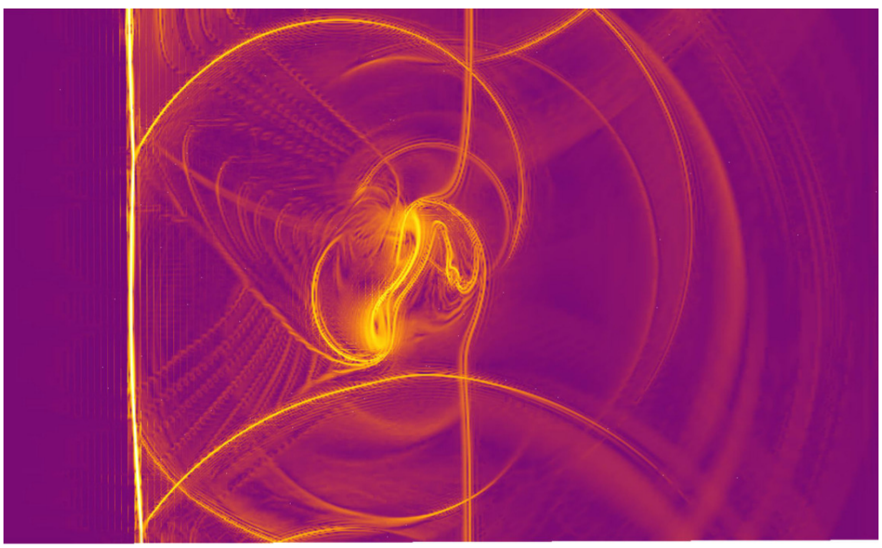

Figure 1: Schlieren representation of the strong-vortex/shock-wave interaction problem at time $t = 0.7$ [s] and Mach number $M = 1.5$ [-] computed with order $p = 6$ and 15000 elements.

High-order simulation methods like Discontinuous Galerkin (DG) have proven suitability for Direct Numerical Simulation (DNS) and (implicit) Large Eddy Simulations (LES) of subsonic flows [1]. In supersonic conditions, it is well known that shock waves may develop. The discontinuity over the shock can not be captured by polynomial interpolation, and therefore both convergence and stability of the simulation deteriorate as Gibbs oscillations develop. In the extreme case, these oscillations lead to unphysical solutions and the failure of the computation. Shock capturing methods (SCM) usually add artificial viscosity to the solution in order to smooth the shock such that it can be safely represented. However, this reduces accuracy and negatively impacts the turbulent kinetic energy budget. It is therefore desirable to reduce its action to a minimum.

The instability of high-order methods, caused by under-interpolation and integration, can be mitigated by leveraging a discrete equivalent of the entropy stability to enhance the conservation and dissipation of energy. Most of these entropy-consistent schemes are based on the use of entropy variables and the "summation-by-parts" (SBP) theorem, leading to the analogue to the integration-by-parts theorem at the discrete level. The vast majority rely on Gauss-Legendre-Lobatto (GLL) nodes that include points on the boundary [2]. This greatly helps for the construction of SBP operators between two elements. However, the GLL quadrature leads to lower integration accuracy and exacerbates the aliasing problem. More recently, entropy stable schemes based on Gauss quadrature nodes, without points on the boundary, have been developed. While these methods improve greatly the accuracy of the solution, such SBP operators are numerically very costly since they introduce an "all-to-all" flux coupling between all degrees of freedom in the element and between two neighbouring elements [3].

This work presents the implementation of a hybrid DG solver for shock capturing. To alleviate the computational cost associated with the entropy stable approach based on Gauss quadrature nodes, this approach is only activated in cells where shock capturing is necessary or the turbulence is under-resolved. Everywhere else, a standard DG formulation is applied. Moreover, if the shock is too strong to be stabilised by this method, artificial viscosity is added. This hybrid approach is compared to standard artificial viscosity methods and full entropy stable schemes.

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