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Model Reduction for Variational Inequalities

I. NIAKH[1,2,3*], A. Ern[2,3], V. Ehrlacher[2,3] and G. Drouet[1]

[1] EDF, 91120 Palaiseau.
[2] CERMICS-École des Ponts, 77455 Marne-la-Vallée cedex 2.
[3] INRIA-Paris, 75589 Paris.
idrissa.niakh@enpc.fr, alexandre.ern@enpc.fr, virginie.ehrlacher@enpc.fr,
guillaume.drouet@edf.fr

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This work deals with model reduction for parametric variational inequalities [2]. As an example of application, we consider the Hertz contact problem. We are interested in the case where the formulation of the problem is dualized by introducing Lagrange multipliers. With the reduced basis method, this formulation leads to a so-called primal-dual strategy: reduced bases are created for each of the variables, primal and dual. The starting point of our work is [1] where the construction of the primal basis and the dual basis is done in a decorrelated way. After sampling primal and dual solutions for parameter values in a training set, Proper Orthogonal Decomposition (POD) was used to compress the primal basis and a Cone Projected Greedy (CPG) algorithm to compress the dual basis. Then the stability of the reduced problem is not guaranteed a priori, since the contact operator associated with the primal and dual bases does not necessarily satisfy an inf-sup condition. In order to satisfy this inf-sup condition, a strategy for constructing the primal basis based on the dual basis was proposed in [2]. The idea is to complete the primal basis with as many functions as there are in the dual basis, each of these functions being determined by a maximization problem to control the corresponding element of the dual basis. In the case where the contact operator is non-parametric, the primal space thus constructed is independent of the parameter so that the completion of the primal basis is computed only once in the *offline* phase. In the case where the contact operator is parameterized, this strategy produces a primal space which depends on the parameter and must therefore be constructed *online*, which considerably reduces the efficiency of the *online* phase. In the present contribution, we propose a strategy to approximate the parametric primal space by a parameter independent space. This space is constructed only once in the *offline* phase, and we establish a sufficient condition to be satisfied during the construction in order to guarantee the inf-sup stability condition for the reduced problem.

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