

An arbitrary Lagrangian-Eulerian formulation for Navier-Stokes flow on deforming surfaces

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A new arbitrary Lagrangian-Eulerian (ALE) formulation for incompressible Navier-Stokes flow on deforming surfaces is presented. The new formulation extends the surface ALE formulation of [1] to more general surface motions. It is based on a new curvilinear surface parameterization that describes the motion of the ALE frame. Its in-plane part becomes fully arbitrary, while its out-of-plane part follows the material motion of the surface. This allows for the description of flows on deforming surfaces using only surface meshes. The unknown fields are the fluid pressure, fluid velocity and surface motion, where the latter two share the same normal velocity. The new theory is implemented in the nonlinear finite element framework of [2] using the stabilization scheme of [3] for the incompressibility constraint. The implementation is verified through several manufactured steady and transient solutions, obtaining near optimal convergence rates in all cases. The new formulation allows for a detailed study of fluidic membranes such as soap films, bubbles and lipid bilayers.

REFERENCES

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