

Nonlinear additive and multiplicative preconditioning strategies for monolithic phase-field fracture models

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One of the state-of-the-art strategies for predicting crack propagation, nucleation, and interaction is the phase-field approach. In this approach, a damage variable is introduced to characterize the material state from intact to fully broken. Although the phase-field approach is a robust tool for predicting crack propagation, it gives rise to strongly nonlinear coupled systems of nonlinear equations that have to be solved at each time step. This is computationally challenging due to a large number of unknowns and the ill-conditioning caused by the local changes in the damage variable.

In this work, we propose to solve the arising nonlinear problems efficiently using additive and multiplicative Schwarz preconditioned inexact Newton's (SPIN) methods [2]. The developed SPIN methods are designed to solve the arising problems in a monolithic manner by taking advantage of the underlying structure of the coupled problem. At each Newton iteration, the SPIN methods construct a nonlinear preconditioner by partitioning the degrees of freedom into two sets, related to the displacement and the phase-field. The underlying nonlinear problem is then solved individually for both fields, in an additive or multiplicative manner. The result of this solution process is used for the construction of the preconditioned coupled linear system, which we solve in order to obtain a search direction.

We will demonstrate the overall convergence behavior of additive and multiplicative SPIN methods using several numerical examples. Moreover, the comparison with the widely-used alternate minimization method will be presented, showing a significant reduction in the number of iterations and the execution time [1].

REFERENCES

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