

A UNIFIED FRAMEWORK TO CONSTRUCT ROBUST HIGH-ORDER DISCONTINUOUS GALERKIN SCHEMES

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We present a unified framework to construct robust high-order accurate discontinuous Galerkin (DG) schemes, where structure-preserving properties and constraints can be imposed using a convex combination of a DG method with a robust lower-order method. Our framework consists of four ingredients: (i) a nodal high-order DG method on Gauss-Lobatto nodes, (ii) a compatible robust subcell Finite Volume (FV) scheme, (iii) a convex combination strategy for the two schemes, which can be element-wise or subcell-wise, and (iv) a strategy to compute the so-called limiting factor, which can be a troubled-cell indicator or a Flux Corrected Transport (FCT) methodology.

We show that, with different choices of the ingredients, we can recover the sparse invariant domain preserving DG scheme of Pazner [3], the entropy-stable (ES) DG method with first-order FV shock capturing of Henneman et al. [1], the ES DG scheme with higher-order FV shock capturing of Rueda-Ramírez et al. [2], among many other possible schemes. We test the robustness of our unified framework with extreme supersonic and hypersonic applications of the Euler, Navier-Stokes, ideal and resistive magnetohydrodynamic equations.

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