

Parameter-robust monolithic solvers for Stokes-Darcy/Biot systems

Wietse M. Boon¹, Martin Hornkjøl², Timo Koch², Miroslav Kuchta³,
Kent-André Mardal² and Ricardo Ruiz-Baier⁴

¹ KTH Royal Institute of Technology, Stockholm, Sweeden; wietse@kth.se

² University of Oslo, Oslo, Norway; {marhorn, timokoch, kent-and}@math.uio.no

³ Simula Research Laboratory, Oslo, Norway; miroslav@simula.no

⁴ Monash University, Melbourne, Australia; ricardo.ruizbaier@monash.edu

Keywords: *Multiphysics problems, Operator preconditioning, Perturbed saddle-point problems*

Systems exhibiting free flow coupled with flow in porous (and deformable) media, as described by coupled Stokes-Darcy/Biot models, arise in numerous environmental, industrial and medical applications. For formulations where the coupling between the components is enforced by the Lagrange multipliers, efficient and parameter-robust solvers of the resulting multiphysics systems, rely on operators in fractional Sobolev spaces defined over the interface [2]. This property is arguably to be expected due to the presence of the explicit coupling variable.

In this talk we discuss formulations of the Stokes-Darcy [1, 3] and Stokes-Biot [4] models which are free of Lagrange multipliers. We establish well-posedness of the systems in a unifying theoretical framework and in turn derive parameter-robust solvers. Despite the absence of the explicit interface variable we show that fractional order operators are again a crucial component for achieving the robustness property.

REFERENCES

- [1] M. Discacciati, E. Migliorini, and A. Quarteroni. Mathematical and numerical models for coupling surface and groundwater flows. *Applied Numerical Mathematics*, 43(1-2):57–74, 2002.
- [2] K. E. Holter, M. Kuchta, and K.-A. Mardal. Robust preconditioning for coupled Stokes–Darcy problems with the Darcy problem in primal form. *Computers & Mathematics with Applications*, 91:56–66, 2021.
- [3] T. Karper, K.-A. Mardal, and R. Winther. Unified finite element discretizations of coupled Darcy–Stokes flow. *Numerical Methods for Partial Differential Equations: An International Journal*, 25(2):311–326, 2009.
- [4] R. Ruiz-Baier, M. Taffetani, H. D. Westermeyer, and I. Yotov. The Biot-Stokes coupling using total pressure: formulation, analysis and application to interfacial flow in the eye. *Computer Methods in Applied Mechanics and Engineering*, 389, 2022.