

# Highly parallel multi-level preconditioners for incompressible flow problems

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Essential parts in bifurcation analysis of flows are continuation of the solution as a function of a parameter and the computation of eigenvalues to check the stability of the solution. The crux of both is the solution of linear systems, which should be robust and fast. In our team two variants of preconditioners are in development: FROSch [2] (<https://shylufrosch.github.io/>) and HYMLS [1] (<https://github.com/nlesc-smcm/hymls>). Both solvers have several things in common: they are based on a domain decomposition approach, make use of parallel linear algebra packages from Trilinos (Xpetra and Epetra, resp.), and allow for acceleration by performing local solves on GPUs. Whereas FROSch implements multilevel Schwarz solvers for general linear systems arising from PDEs, HYMLS is a special purpose multilevel solver for finite volume methods on structured grids for solving the incompressible Navier-Stokes equations extended with transport equations. HYLMS always iterates in the divergence free space, meaning that its solution is always exactly conserving mass; the same can be achieved with specific settings for FROSch. This property is important for robustness.

We also implemented in Python the package fvm (<https://github.com/BIMAU/fvm>), which contains a set of standard CFD problems and a continuation code. Moreover, we built in Python package jadapy (<https://github.com/BIMAU/jadapy>), which enables us to compute eigenvalues with the Jacobi-Davidson method. Both packages have interfaces to the C++ package HYMLS.

During the presentation, we will discuss the design of the solver packages and their parallel scalability up to thousands of cores for large scale problems. Moreover, we will compare these methods against each other and against other methods available in Trilinos.

## REFERENCES

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