

Variational space-time localization for adjoint-based adaptivity of Navier-Stokes equations

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In this talk, we will present an extension of the variational partition of unity (PU) approach [3] for the localization of global error estimators obtained by a dual weighted residual (DWR) method [1] in a space-time setting. As model problem, we apply the PU-DWR method to the nonstationary Stokes and Navier-Stokes equations. Starting from space-time formulations, we derive the global DWR estimators and the corresponding adjoint problem. The latter is linear, but is running backward in time. The focus will then be on the application of the PU localization of this global error estimator for adaptive refinement [4]. Finally, we will show some results based on the tensor-product space-time software introduced in [2]. In some numerical tests, mesh refinement studies in time and space as well as effectivity indices are adopted to substantiate our framework. This work serves as preparatory work for fluid-structure interactions in which the incompressible Navier-Stokes equations are coupled with nonlinear elastodynamics.

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