Locally refined spline spaces – Properties and structureS for different refinement frameworks

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ABSTRACT

Isogeometric analysis (IgA) uses spline spaces rather than finite element spaces for solving PDEs. When IgA was introduced in 2005 [1] the natural choice was to build on tensor product B-spline spaces already known from Computer Aided Design (CAD). However, it was soon evident that IgA needed spline technology that allowed adding degrees of freedom in specific geometric areas, i.e., support of local refinement. The first such technology, Hierarchical B-splines (HB) [2] had already been introduced in 1989. HB is based on selecting tensor product B-splines from a collection of nested spline spaces. These spline spaces are defined by a nested sequence of grids determined by scaled lattices. In 2003 T-splines (TS) [3] were introduced where the refinement is performed in the mesh of control vertices, the T-mesh. The increased interest in local refinement triggered further research: Truncated Hierarchical B-splines (THB) [4] in 2012 add partition of unity to HB, and the truncation mechanism was later introduced to Catmull-Clark subdivision [5] and T-splines [6]; Locally Refined B-splines (LRB) [7] in 2013 generalize refinement of univariate spline spaces by knot insertion to the multivariate case. The starting point of LRB is as for TS a tensor product spline space. Knot insertion is replaced by insertion of mesh-rectangles into the LR-mesh, the mesh that describes the piecewise structure of the spline space. Yet another approach is to define a mesh with T-joints and use this as the basis for defining the spline space [8] in 2008.

The above approaches for local refinement have all been proved as good spline technologies for IgA. Little effort has been invested until now to understand the differences and similarities of the approaches. The focus of the mini-symposium is to provide a deeper understanding of:

* The structure of the spline space generated by the different approaches.
* For TS and LRB there are cases when the resulting collection of tensor product B-splines spanning the spline space do not form a basis. How restrictive are these conditions for ensuring linear independence?
* Assuming conditions for linear independences are met for TS and LRB. Do the different approaches impose patterns or specific structures on the basis used for spanning the spline space?
* Are some of the approaches more flexible than other approaches with respect to certain uses?
* Can the spline space of the different approaches be reproduced by the other approaches, or be a subset of spline spaces of the other approaches?
* Do these differences matter?

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